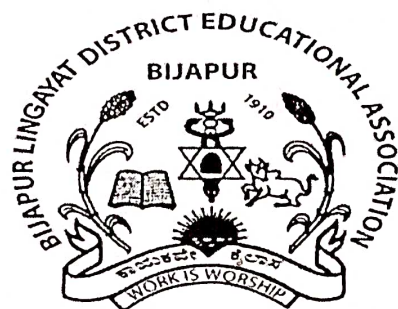


**BLDE ASSOCIATION'S**  
**S.B.ARTS AND K.C.P SCIENCE COLLEGE,**  
**VIJAYAPUR**



**DEPARTMENT OF MATHEMATICS**  
**ADVANCED LEARNERS SEMINAR REPORT**  
**2022-23 (EVEN SEMESTER)**



B. L. D. E. ASSOCIATION'S

S. B. ARTS AND K. C. P. SCIENCE COLLEGE  
VIJAYAPUR



**DEPARTMENT OF MATHEMATICS**

Date: 18/07/2023

**NOTICE**

The UG Department of Mathematics will be conducting seminar for the B.Sc II, IV, VI Semester students (Advance learners) of the Academic year 2022-23(Even Semester) from 24/07/2023 to 01/08/2023 during regular classes.

  
**HOD**

H. O. D:  
Department of Mathematics,  
S. B. Arts & K. C. P. Science  
College, VIJAYAPUR.

  
**IQAC Coordinator**

**IQAC, Co-ordinator**  
S.B.Arts & K.C.P.Science College,  
Vijayapur.

  
**Principal**

Principal,  
S.B. Arts and KCP Science College  
VIJAYAPUR


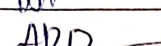
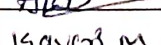
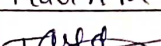
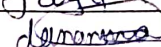
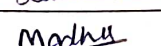
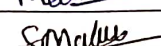
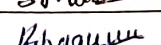
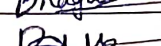
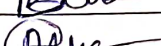
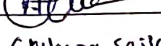
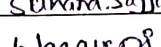
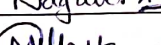
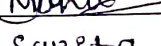
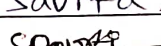
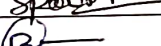
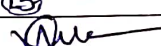
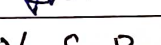
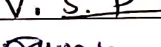

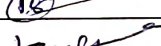
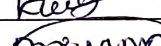
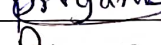
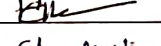
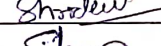
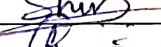
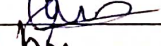

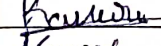
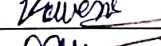
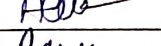
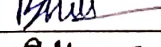
### **Guidelines for identifying Advanced Learners (bright Students):**

Advanced Learners are identified based on the first internal examinations those who secured greater than 80%.

### **Following Special activities are conducted for Advanced Learners:**

1. Guiding for carrier planning.
2. Discussion of seminar on the advanced topic.
3. Guiding and encouraging communicating research papers in conference /Journals.
4. Encouraging participating in various symposiums like quiz, conference etc.

**List of Advance Learners with Signature**

Sl.No	Name of the student	Class	Reg.No	Signature
1.	NINGARADDY	B.Sc II	U15KM22S0137	
2.	ABDUL YADAGIRI	B.Sc II	U15KM22S0221	
3.	KAVERI MELINAKERI	B.Sc II	U15KM22S0001	
4.	JAYASHREE	B.Sc II	U15KM22S0153	
5.	DANAMMA POOJARI	B.Sc II	U15KM22S0294	
6.	MADHUMATI KALAGOND	B.Sc II	U15KM22S0024	
7.	SAHANA MALLAPPAGOL	B.Sc II	U15KM22S0125	
8.	BHAGYASHREE JAMBRE	B.Sc II	U15KM22S0128	
9.	BHAGYASHREE MATHAPATI	B.Sc II	U15KM22S0022	
10.	AKSHATA RAMESH BADAGI	B.Sc II	U15KM22S0162	
11.	SAHANA SAJJAN	B.Sc II	U15KM22S0224	
12.	NAGAVENI HARWAL	B.Sc IV	U15KM21S0389	
13.	MADHU BARATAGI	B.Sc IV	U15KM21S0471	
14.	SAVITA BILIJADAR	B.Sc IV	U15KM21S0366	
15.	SPOORTI SIDDAPUR	B.Sc IV	U15KM21S0372	
16.	BHAGYANIDHI BUDIHAL	B.Sc IV	U15KM21S0339	
17.	VIDYA BIRADAR	B.Sc IV	U15KM21S0396	
18.	VISHAL SARUR	B.Sc IV	U15KM21S0412	
19.	RAGINI DWIVEDI	B.Sc IV	U15KM21S0506	
20.	BHAGYASHREE MYAGERI	B.Sc IV	U15KM21S0154	
21.	KASHIBAI PUJARI	B.Sc IV	U15KM21S0370	
22.	PRIYANKA	B.Sc IV	U15KM21S0189	
23.	PALLAVI BHUYAR	B.Sc VI	S2028665	
24.	SHRIDEVI HALLI	B.Sc VI	S2028819	
25.	ANUSHA BAGEWADI	B.Sc VI	S2028475	
26.	GANESH MALI	B.Sc VI	S2028552	
27.	AMBIKA MIRAGI	B.Sc VI	S2028459	
28.	KAVERI AKKALAKOT	B.Sc VI	S2028587	
29.	KAVERI KOLI	B.Sc VI	S2028586	
30.	ASHWINI HADIMANI	B.Sc VI	S2028498	
31.	BHAGYASHREE BIRADAR	B.Sc VI	S2028514	
32.	AISHWARYA HIREMATH	B.Sc VI	S2028423	



## Seminar Topics for Advance Learners

S.No	Name of the student	Seminar topics	Date
1	NINGARADDY	Cayley Hamilton theorem	24-07-2023
2	ABDUL YADAGIRI	Jacobian and Examples	24-07-2023
3	KAVERI MELINAKERI	System of Linear equations	24-07-2023
4	JAYASHREE	Roll's theorem	24-07-2023
5	DANAMMA POOJARI	Cauchy mean value theorem.	24-07-2023
6	MADHUMATI KALAGOND	Types of matrices.	25-07-2023
7	SAHANA MALLAPPAGOL	Intermediate value theorem	25-07-2023
8	BHAGYASHREE JAMBRE	Lagrange's mean value theorem.	25-07-2023
9	BHAGYASHREE MATHAPATI	Taylor's theorem.	25-07-2023
10	AKSHATA BADAGI	Cauchy's root test	25-07-2023
11	SAHANA SAJJAN	Groups and examples	25-07-2023
12	NAGAVENI HARWAL	Periodic functions	25-07-2023
13	MADHU BARATAGI	Fourier series	26-07-2023
14	SAVITA BILIJADAR	Laplace transform of integral form	26-07-2023
15	SPOORTI SIDDAPUR	Dirac delta function	26-07-2023
16	BHAGYANIDHI BUDIHAL	Shifting property	26-07-2023
17	VIDYA BIRADAR	Limit definition and examples	26-07-2023
18	VISHAL SARUR	Fourier series of even and odd functions	24-07-2023
19	RAGINI DWIVEDI	Sequences, types of sequences	24-07-2023
20	BHAGYASHREE MYAGERI	Monotonic sequences	24-07-2023
21	KASHIBAI PUJARI	Definition of Group	24-07-2023
22	PRIYANKA	Properties of groups	24-07-2023
23	PALLAVI BHUYAR	Cauchys integral formula	28-07-2023
24	SHRIDEVI HALLI	Cauchys Theroem	28-07-2023
25	ANUSHA BAGEWAD	Laurent's Series	28-07-2023
26	GANESH MALI	Charpits Method	28-07-2023
27	AMBIKA MIRAGI	Laurent's Series	24-07-2023
28	KAVERI AKKALAKOT	Condition for R-intergrable	24-07-2023
29	KAVERI KOLI	Prove that analytic function with constant modulus is constant	24-07-2023
30	ASHWINI HADIMANI	Cauchys Theroem	24-07-2023
31	BHAGYASHREE BIRADAR	Examples on double integration	24-07-2023
32	AISHWARYA HIREMATH	Necessary and Sufficient condition for R-intergrable	24-07-2023



B. L. D. E. ASSOCIATION'S

S. B. ARTS AND K. C. P. SCIENCE COLLEGE  
VIJAYAPUR



**DEPARTMENT OF MATHEMATICS**

Date: 01/08/2023


**Seminar Report**

The Department of Mathematics conducted the seminars for Advance Learners of B.Sc II, IV, and VI semester. Students given seminars on the chapters covered in their syllabus, each student given 10 minutes for the topic which covers 8 minutes for seminar and 2 minutes for discussion.


The objective of conducting the seminars was to help the students that to improve their communication skills, knowledge and to reduce stage fear for the students.

  
**HOD**

H. O. D:  
Department of Mathematics,  
S. B. Arts & K. C. P. Science  
College, VIJAYAPUR.

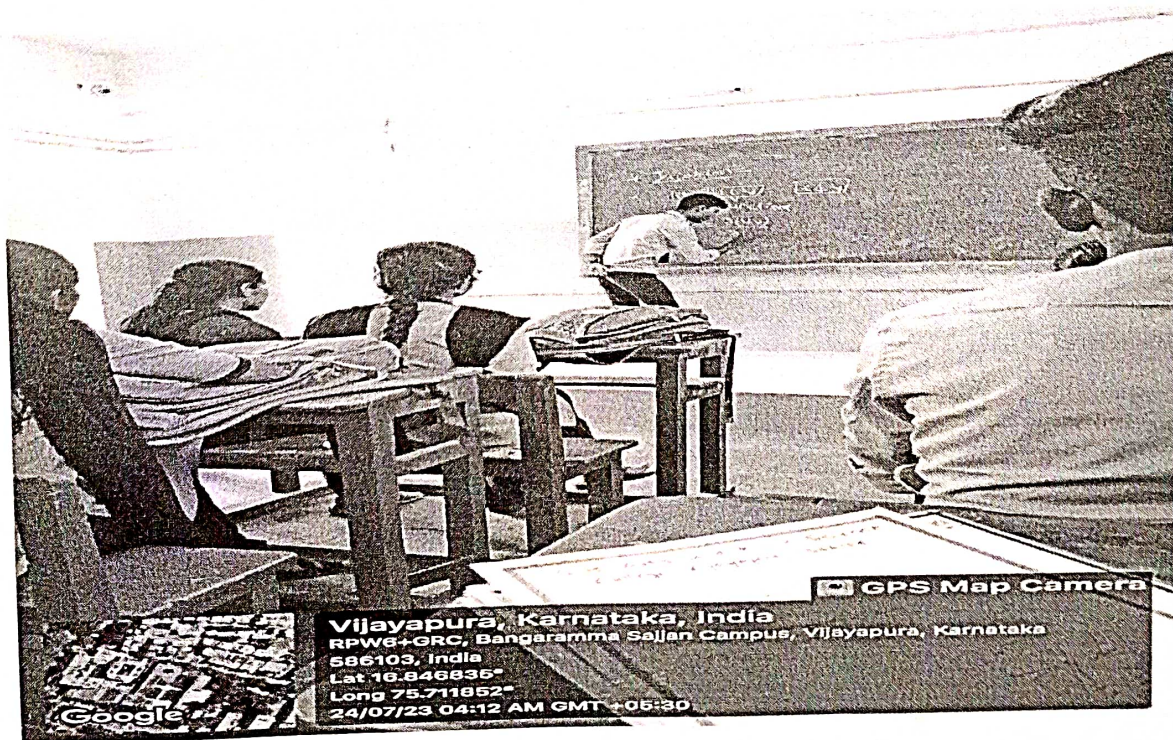
  
**IQAC Coordinator**

**IQAC, Co-ordinator**  
S.B.Arts & K.C.P.Science College,  
Vijayapur.

  
**Principal**

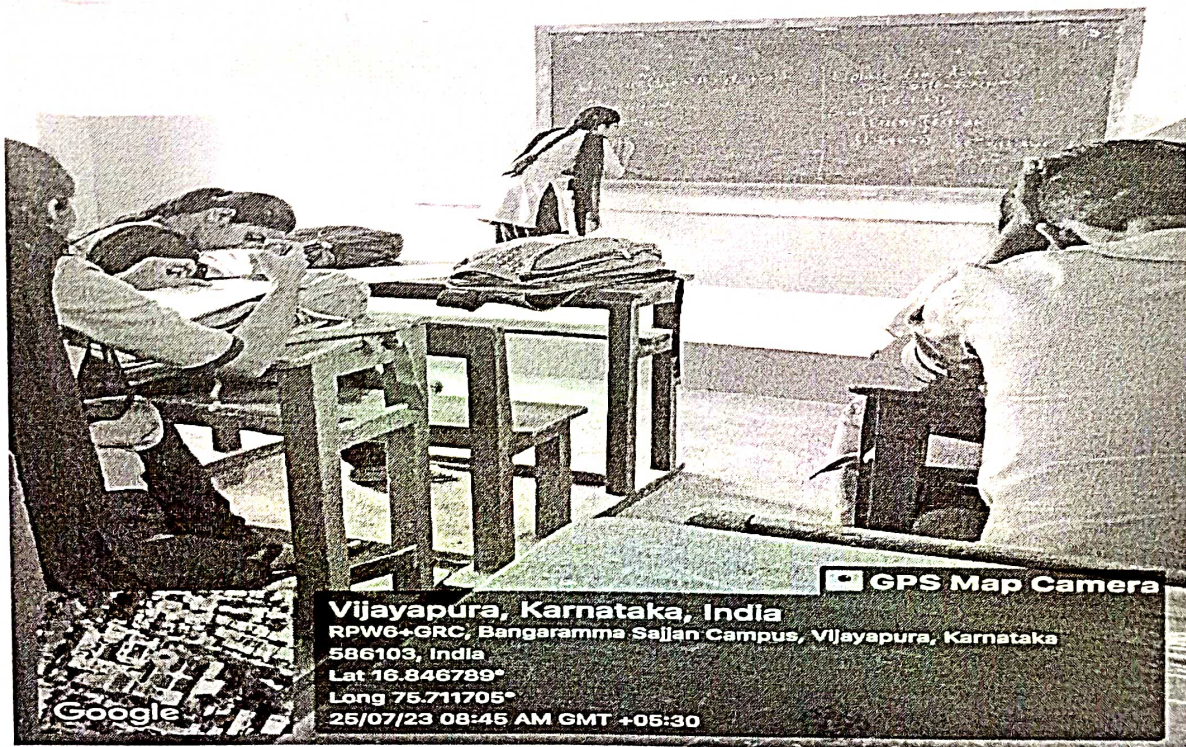
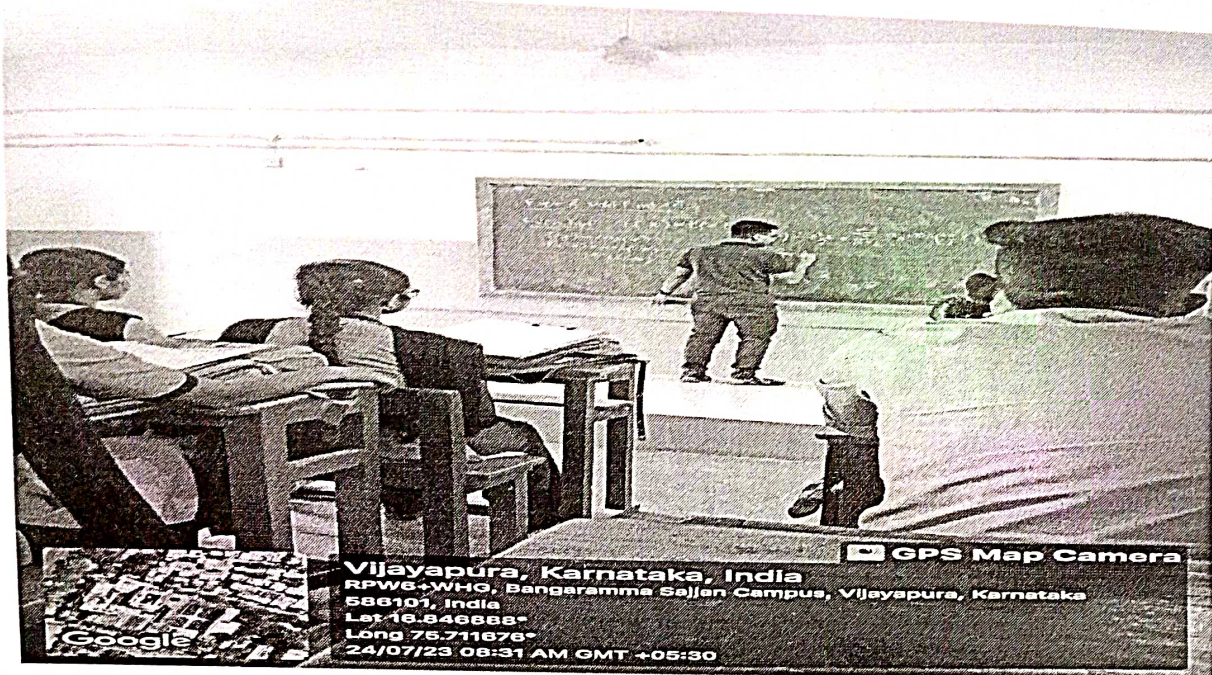
Principal,  
S.B. Arts and KCP Science College  
VIJAYAPUR





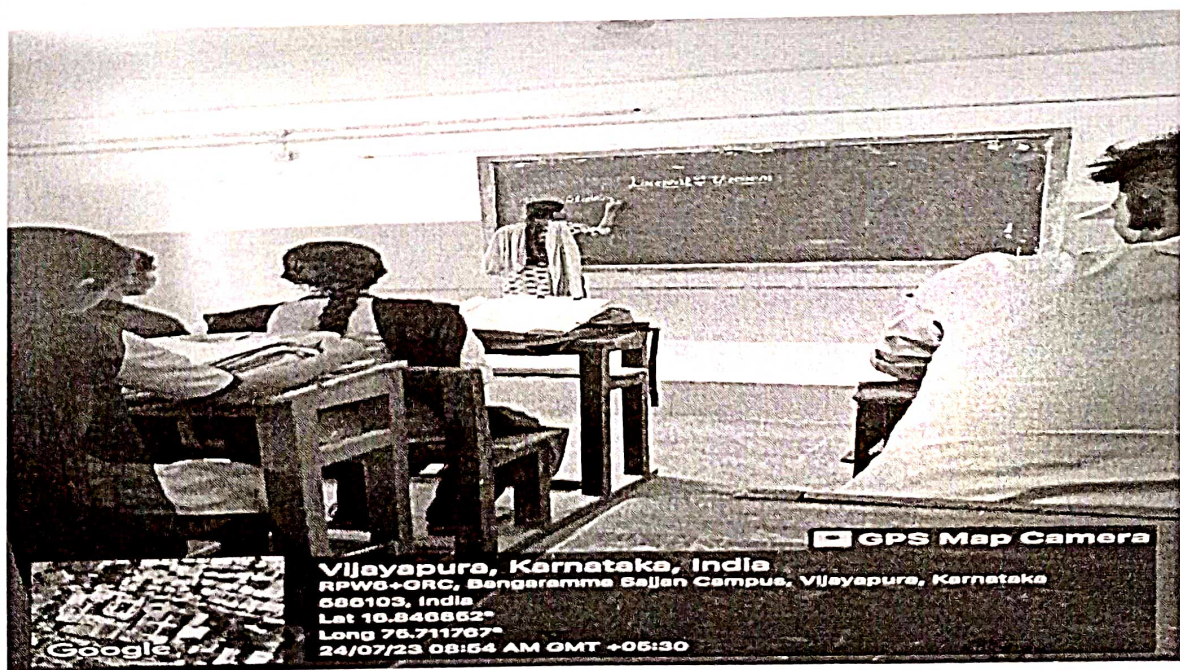
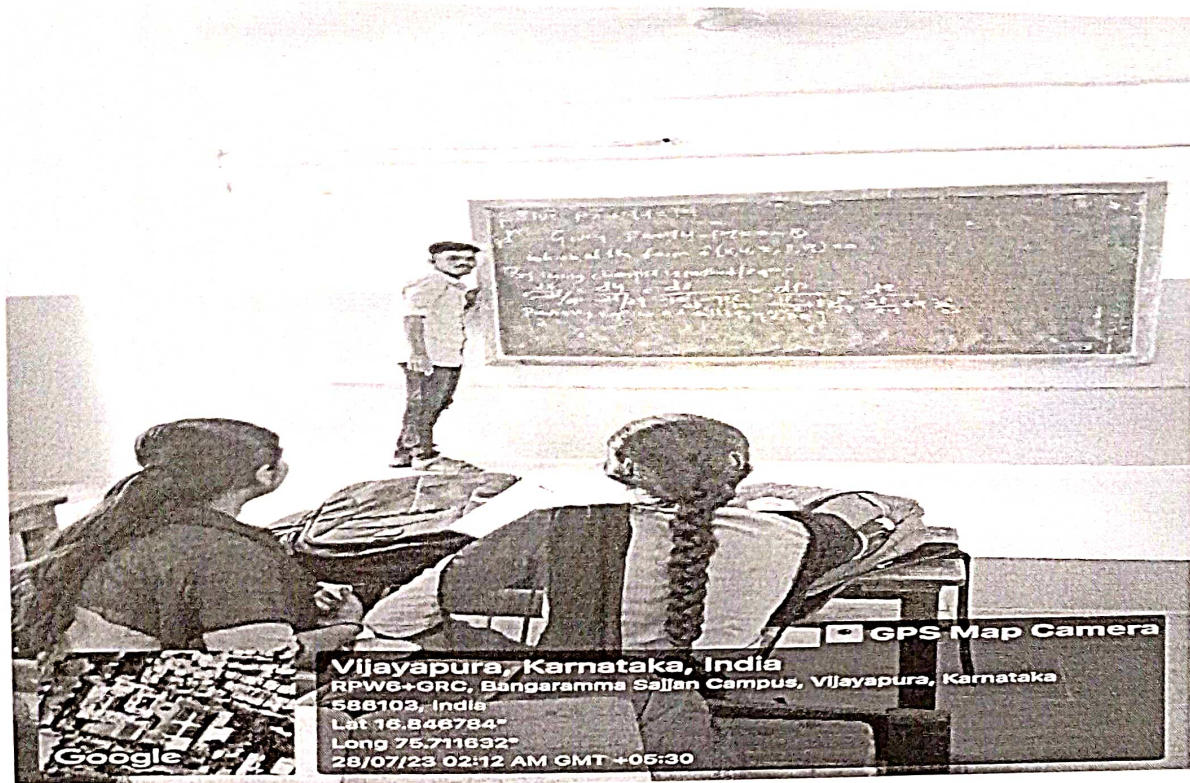
**Advanced learner students giving their seminars**





**Advanced learner students giving their seminars**





## Advanced learner students giving their seminars

*[Signature]*  
 O. D.

Department of Mathematics,  
 S. b. Arts & K. C. P. Science  
 College, BILAPUR.

*[Signature]*

**IQAC, Co-ordinator**  
 S.B.Arts & K.C.P.Science College,  
 Vijayapur.

*[Signature]*

**Principal,**  
 S.B. Arts and KCP Science College  
 VIJAYAPUR



# S B ARTS & KCP SCIENCE COLLEGE VIJAYAPUR

Name: Ganesh. Mali.

Subject: → Mathematics

Roll no: → S2028552. ~~VI~~ Semester

Topic name: → Charpit's Method.

Guided by: → Laxmi patil. mam

Class: → BSc VI sem

Solve  $Px + Qy = Pq$  by Charpit's Method.

Sol<sup>n</sup>: Given differential equation is

$$Px + Qy = Pq \quad \text{OR} \quad Px + Qy - Pq = 0 \quad \text{--- (1)}$$

which of the form  $f(x, y, z, P, Q) = 0$  then

By Charpit's method, we have.

$$\frac{dx}{-\frac{\partial f}{\partial P}} = \frac{dy}{-\frac{\partial f}{\partial Q}} = \frac{dz}{P\frac{\partial f}{\partial P} - Q\frac{\partial f}{\partial Q}} = \frac{dP}{\frac{\partial f}{\partial x} + P\frac{\partial f}{\partial z}} = \frac{dQ}{\frac{\partial f}{\partial y} + Q\frac{\partial f}{\partial z}}$$

Partially differentiating eq<sup>n</sup> (1) w.r.t  $x, y, z, P, Q$

$$\frac{\partial f}{\partial x} = P, \quad \frac{\partial f}{\partial y} = Q, \quad \frac{\partial f}{\partial z} = 0$$

$$\frac{\partial f}{\partial P} = x - Q, \quad \frac{\partial f}{\partial Q} = y - P$$

$$\frac{dx}{-(x-Q)} = \frac{dy}{-(y-P)} = \frac{dz}{-P(x-Q) - Q(y-P)} = \frac{dP}{P+P(0)} = \frac{dQ}{Q+Q(0)}$$

$$\frac{dx}{Q-x} = \frac{dy}{P-y} = \frac{dz}{-P(x-Q) - Q(y-P)} = \frac{dP}{P} = \frac{dQ}{Q}$$

Taking last two fractions then integrating, we get

$$\int \frac{dP}{P} = \int \frac{dQ}{Q}$$

$$\log p = \log q + \log a.$$

$$\log p - \log q = \log a$$

$$\log(p/q) = \log a$$

$$p/q = a$$

$$p = qa \text{ --- (2)}$$

From eqn (2) & (1) we get

$$px + qy - pq = 0$$

$$aqx + qy = aqq$$

$$x(ax+y) = aqx$$

$$(ax+y) = aq$$

$$q = \frac{ax+y}{a}$$

$$p = a \left( \frac{ax+y}{a} \right)$$

$$p = ax+y$$

Now,  $dz = p dx + q dy$

$$dz = (ax+y) dx + \left( \frac{ax+y}{a} \right) dx$$

$$dz = (ax+y) \left( dx + \frac{1}{a} dy \right)$$

$$dz = (ax+y) \left[ \frac{adx+dy}{a} \right]$$



$$a dz = (ax+y)(adx+dy)$$

On integrating, we get

$$a \int dz = \int (ax+y)(adx+dy)$$

$$\text{put } (ax+y) = z$$

$$adx+dy = dz$$

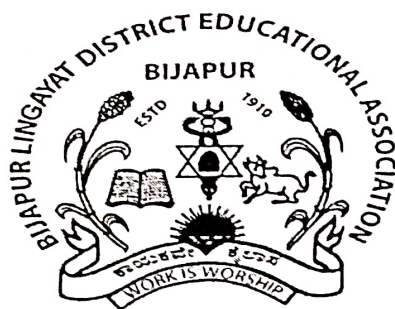
$$a z = \int z dz$$

$$a z = \frac{z^2}{2} + C_1$$

$$a z = \frac{(ax+y)^2}{2} + C_1$$

$$\left[ z = \frac{(ax+y)^2}{2a} + C \right] //$$

**BLDE ASSOCIATION'S**  
**S.B.ARTS AND K.C.P SCIENCE COLLEGE,**  
**VIJAYAPUR**



**DEPARTMENT OF MATHEMATICS**  
**ADVANCED LEARNERS SEMINAR REPORT**  
**2022-23 (ODD SEMESTER)**



B. L. D. E. ASSOCIATION'S  
S. B. ARTS AND K. C. P. SCIENCE COLLEGE  
VIJAYAPUR



**DEPARTMENT OF MATHEMATICS**

Date: 02/01/2023

**NOTICE**

The UG Department of Mathematics will be conducting seminar for the B.Sc I, III, V Semester students (Advance learners) of the Academic year 2022-23(Odd Semester) from 05/01/2023 to 12/02/2023 during regular classes.

**HOD**

H. O. D.

Department of Mathematics,  
S. B. Arts & K. C. P. Science  
College, VIJAYAPUR.

**IQAC Coordinator**

**IQAC, Co-ordinator**  
S.B.Arts & K.C.P.Science College  
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4. Encouraging to participate in various symposium like quiz, conference etc.



**List of Advance Learners with Signature**

Sl.No	Name of the student	Class	Reg.No	Signature
1.	KAVERI MELINAKERI	B.Sc I	U15KM22S0001	(Kaveri)
2.	BHAGYASHREE MATHAPATI	B.Sc I	U15KM22S0022	(Bhagya)
3.	JAYASHREE	B.Sc I	U15KM22S0153	(Jayashree)
4.	DANAMMA POOJARI	B.Sc I	U15KM22S0294	(Danamma)
5.	SAKSHI AJUR	B.Sc I	U15KM22S0005	(Sakshi)
6.	SUSHMITA JANGAMASHETTI	B.Sc I	U15KM22S0026	S.P. Sangamashetti
7.	NINGARADDY	B.Sc I	U15KM22S0137	(Ningarddy)
8.	BHOOMIKA KONDAGULI	B.Sc I	U15KM22S0180	(Bhoomika)
9.	ROOPA PUJARI	B.Sc I	U15KM22S0131	(Roopa)
10.	ABDUL YADAGIRI	B.Sc I	U15KM22S0221	(A.Y.)
11.	SAHANA SAJJAN	B.Sc I	U15KM22S0224	(Sahana)
12.	PRIYANKA	B.Sc III	U15KM21S0189	(Priyanka)
13.	BHAGYANIDHI BUDIHAI	B.Sc III	U15KM21S0339	(Bhagya)
14.	SUVARNA YALAWAR	B.Sc III	U15KM21S0357	(Suvarna)
15.	MADHU BARATAGI	B.Sc III	U15KM21S0471	(Madhu)
16.	RAGINI DWIVEDI	B.Sc III	U15KM21S0506	(Ragini)
17.	BHUVANESHWARI AGASAR	B.Sc III	U15KM21S0097	(Bhuvaneswari)
18.	NIKHITA VIJAPUR	B.Sc III	U15KM21S0130	N. S. Vijapur
19.	NIKHEETA PAWAR	B.Sc III	U15KM21S0148	(Nikheeta)
20.	BHAGYASHREE MYAGERI	B.Sc III	U15KM21S0154	(Bhagya)
21.	KASHIBAI PUJARI	B.Sc III	U15KM21S0370	(Kashibai)
22.	SPOORTI SIDDAPUR	B.Sc III	U15KM21S0372	(Spoorti)
23.	ARCHANA MARUTI DAPTARADAR	B.Sc V	S2028486	A.M. Daptaradar
24.	ARPITA ARAVIND KORI	B.Sc V	S2028487	A.A. Kori
25.	ASHWINI KSHATRI	B.Sc V	S2028492	A.K. Shatri
26.	ASHWINI DONI	B.Sc V	S2028494	(Ashwini)
27.	BHAGYA H BIRADAR	B.Sc V	S2028507	(Bhagya)
28.	BHAGYASHREE	B.Sc V	S2028509	B. Nagabhatti
29.	BHAGYASHREE BIRADAR	B.Sc V	S2028514	(Bhagya)
30.	BHAGYASHREE GUMASHETTI	B.Sc V	S2028516	(Bhagya)
31.	BHAGYASHREE TENIHALLI	B.Sc V	S2028524	(Bhagya)
32.	BHAVANI BASAYYA MUTTAGI	B.Sc V	S2028526	(Bhavani)
33.	BHAVANI PRAKASH KULKARNI	B.Sc V	S2028527	(Bhavani)

### Seminar Topics for Advance Learners

S.No	Name of the student	Seminar topics	Date
1	KAVERI MELINAKERI	Cayley Hamilton theorem, matrices and types of matrices.	05-01-2023
2	BHAGYASHREE MATHAPATI	Limit and Continuity, Cauchy mean value theorem.	05-01-2023
3	JAYASHREE	Polar Co-ordinates, Relation between Polar and Cartesian Co-ordinates.	05-01-2023
4	DANAMMA POOJARI	Angle between radius vector and tangent vector, examples.	05-01-2023
5	SAKSHI AJUR	Row and Column reduction to Echelon form.	05-01-2023
6	SUSHMITA JANGAMASHETTI	Derivative of an arc in Cartesian parametric polar forms.	06-01-2023
7	NINGARADDY	Intermediate value theorem.	06-01-2023
8	BHOOMIKA KONDAGULI	Roll's theorem and examples.	06-01-2023
9	ROOPA PUJARI	Lagrange's mean value theorem.	06-01-2023
10	ABDUL YADAGIRI	Taylor's theorem.	06-01-2023
11	SAHANA SAJJAN	Leibnitz theorem and its applications.	12-01-2023
12	PRIYANKA	Sequences, types of sequences and Algebra limits.	05-01-2023
13	BHAGYANIDHI BUDIHAL	Bernoulli's equation and exact differential equation.	05-01-2023
14	SUVARNA YALAWAR	Necessary and sufficient condition for exact differential equation.	05-01-2023
15	MADHU BARATAGI	D'Alemberts ratio test and Cauchy's root test.	05-01-2023
16	RAGINI DWIVEDI	Cauchy's first and second theorems on limits.	05-01-2023
17	BHUVANESHWARI AGASAR	Necessary and sufficient condition for extreme value.	06-01-2023
18	NIKHITA VIJAPUR	Cauchy's first and second theorems on limits.	06-01-2023
19	NIKHEETA PAWAR	Clairaut's equations reducible to Clairaut's form.	06-01-2023



20	BHAGYASHREE MYAGERI	Differential equations of first order higher degree Cyclic groups and its properties.	06-01-2023
21	KASHIBAI PUJARI	Theorems on monotonic sequences.	06-01-2023
22	SPOORTI SIDDAPUR	Properties of groups.	
23	ARCHANA DAPTARADAR	Partition of $[a,b]$ , Norm of partition, refinement of partition.	06-01-2023
24	ARPITA KORI	upper and lower Riemann sum Solution of differential equation by finding a suitable integrating factor.	06-01-2023
25	ASHWINI KSHATRI	Solution of differential equation by finding a suitable integrating factor.	06-01-2023
26	ASHWINI DONI	Theorems on Riemann integration.	06-01-2023
27	BHAGYA H BIRADAR	The first and second mean value theorem of integral calculus.	06-01-2023
28	BHAGYASHREE	Relation between Beeta and Gamma functions.	06-01-2023
29	BHAGYASHREE BIRADAR	Double and Triple integral.	07-01-2023
30	BHAGYASHREE GUMASHETTI	Bisection method and Iteration method.	07-01-2023
31	BHAGYASHREE TENIHALLI	Newton forward and backward difference.	07-01-2023
32	BHAVANI MUTTAGI	Picard's and Range Kutta method of order two.	07-01-2023
33	BHAVANI KULKARNI	Necessary and sufficient condition for Riemann integration.	07-01-2023

H. O. D.

Department of Mathematics,  
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**IQAC, Co-ordinator**  
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**Principal,**  
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B. L. D. E. ASSOCIATION'S

S. B. ARTS AND K. C. P. SCIENCE COLLEGE  
VIJAYAPUR



DEPARTMENT OF MATHEMATICS

Date: 21/01/2023

## Seminar Report

The Department of Mathematics conducted the seminars for Advance Learners of B.Sc I,III,V semester. Students given seminars on the chapters covered in their syllabus, each student given 10 minutes for the topic which covers 8 minutes for seminar and 2 minutes for discussion.

The objective of conducting the seminars was to help the students that to improve their communication skills, knowledge and to reduce stage fear for the students.

HOD

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Department of Mathematics,  
S. B. Arts & K. C. P. Science  
College, VIJAYAPUR.

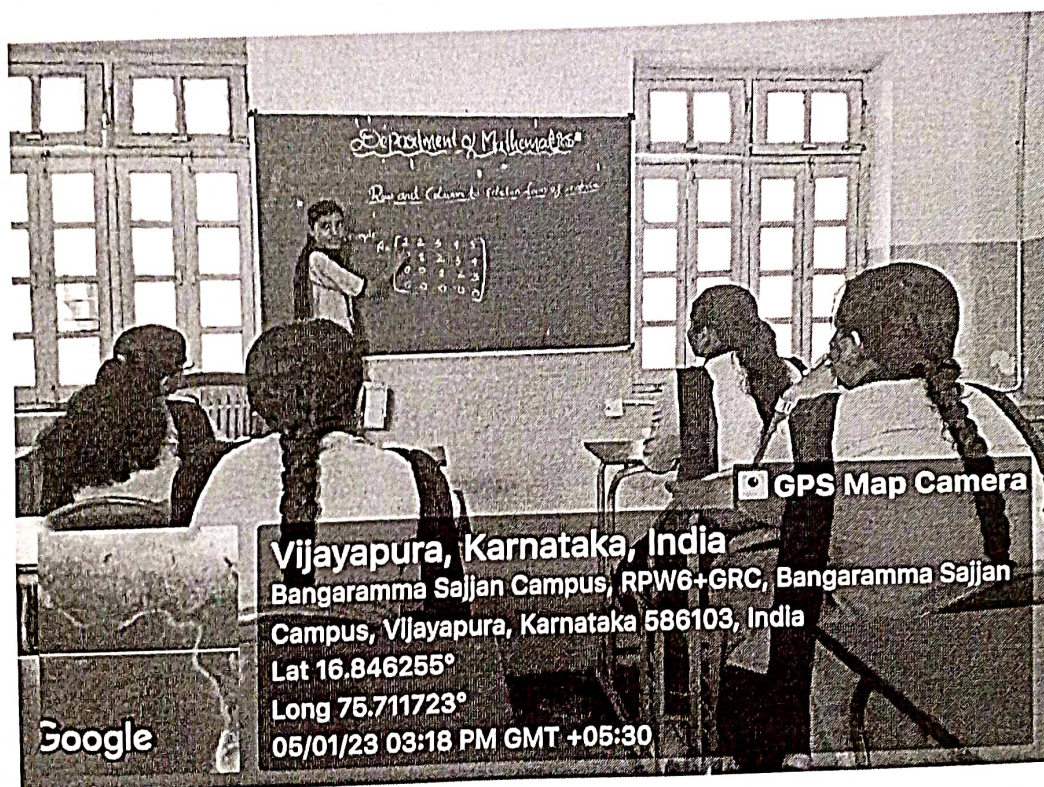
IQAC Coordinator

IQAC, Co-ordinator  
S.B.Arts & K.C.P.Science College,  
Vijayapur.

Principal  
Principal,

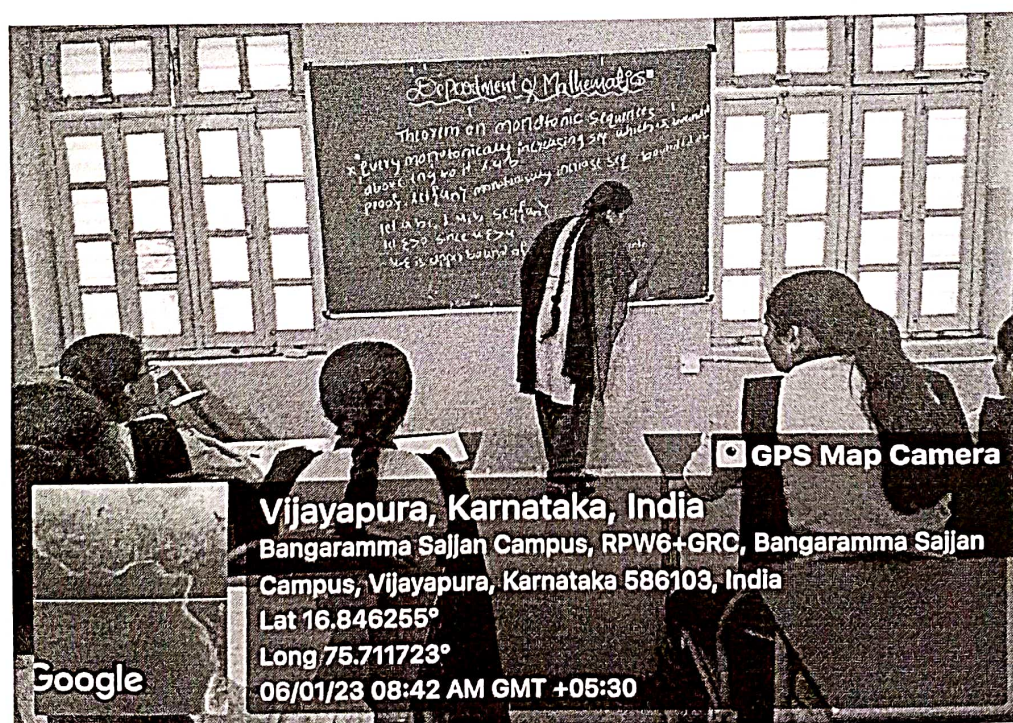
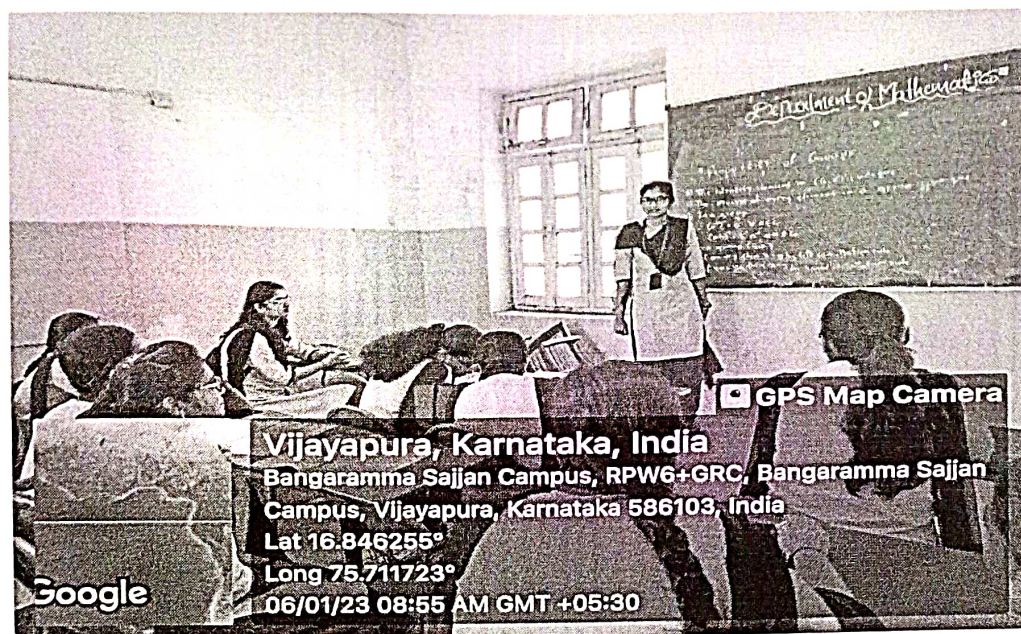
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**Advanced learner students giving their seminars**





**Advanced learner students giving their seminars**





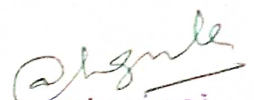
## Advanced learner students giving their seminars

  
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Department of Mathematics,  
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## SEMINAR

Name: Bhoomika M. Kondaguli.

Subject: Mathematics.

Class: B.Sc.II<sup>nd</sup> Sem., Roll.No: 128.

UUCMS: U15KM2250180.

Topic: Rolle's theorem and its  
examples.

Signature:

# Rolle's theorem:

Statement: Let  $f(x)$  be a function that satisfies the following three hypothesis -

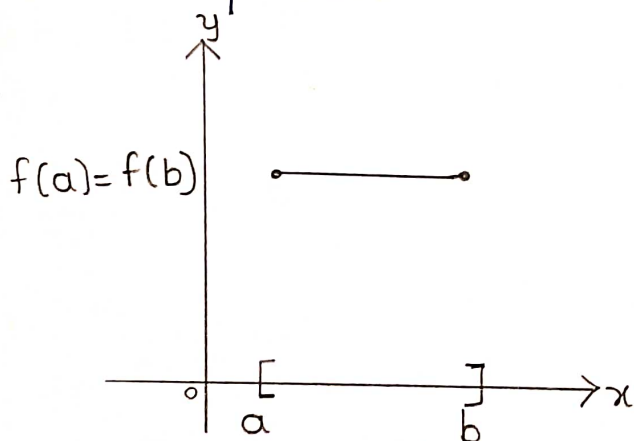
- i)  $f(x)$  is continuous on  $[a, b]$
- ii)  $f(x)$  is differentiable on  $(a, b)$
- iii)  $f(a) = f(b)$

then there exist atleast one point  $C \in (a, b)$  such that  $f'(C) = 0$ .

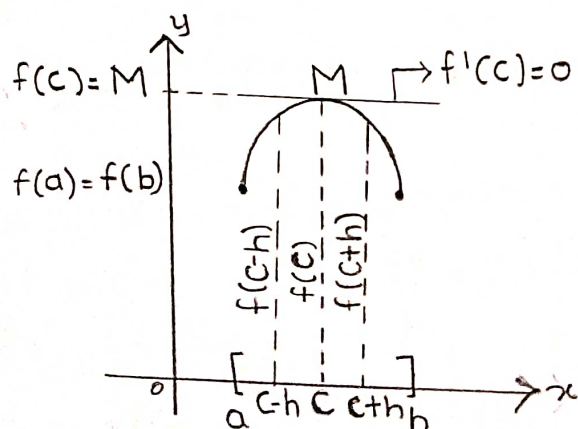
proof - Given function  $f(x)$  is continuous on  $[a, b]$  then  $f(x)$  attains its supremum and infimum - atleast once.

There are two possibilities -

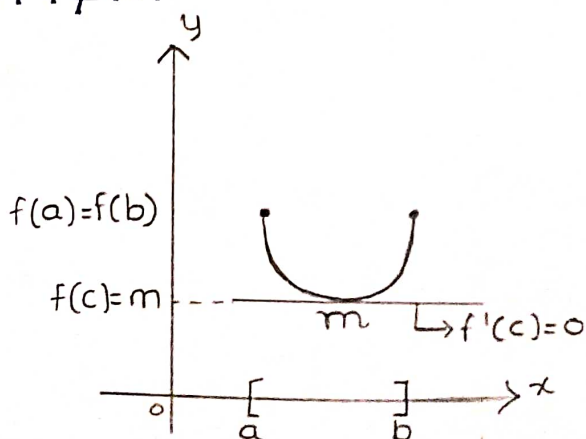
(i)  $M = m$



(ii)  $M \neq m$



(iii)  $M \neq m$





Case-I :  $M=m$  on closed interval  $[a, b]$

From fig. i -  $f(x) = K$

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow f'(c) = 0$$

Case-III :  $M \neq m$  on  $[a, b]$  then atleast one of the bounds  $M$  and  $m$  is different from  $f(a)$  and  $f(b)$  [fig. ii and fig. iii]

$$f(a) \neq f(b) \neq M$$

then there exist  $c \in (a, b)$ , such that  $f(c) = M$   
i.e.,  $f(x)$  is maximum at  $c$ .

$$f(c-h) \leq f(c) \rightarrow \textcircled{1}$$

$$f(c+h) \leq f(c) \rightarrow \textcircled{2}$$

$$\Rightarrow f(c-h) - f(c) \leq 0$$

$$f(c+h) - f(c) \leq 0 \quad \text{where } h \text{ is small}$$

$$\Rightarrow \frac{f(c+h) - f(c)}{h} \leq 0$$

$$\frac{f(c-h) - f(c)}{-h} \geq 0$$

Taking limit as  $h \rightarrow 0$ ,

$$\lim_{h \rightarrow 0} \frac{f(c-h) - f(c)}{-h} \geq 0 \Rightarrow Lf'(c) \geq 0$$

$$\lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} \leq 0 \Rightarrow Rf'(c) \leq 0$$

Since,  $f(x)$  is differentiable at  $c \in (a, b)$ , so we write

$$Lf'(c) = Rf'(c) = 0$$

$$f'(c) = 0$$

Hence, there is a point  $c \in (a, b)$  at which  $f'(c) = 0$  //



## Examples on Rolle's theorem:

1) Verify Rolle's theorem for  $f(x) = x^2 - 6x + 8$  in  $[2, 4]$ .

Solution: Given  $f(x) = x^2 - 6x + 8$  in  $[2, 4]$ .

i) Since  $f(x) = x^2 - 6x + 8$  is a polynomial function in  $x$

$\Rightarrow$  It is continuous for every value of  $x$ .

ii) Derivative  $f'(x) = 2x - 6 \quad \forall x \in (2, 4)$

$\Rightarrow f(x)$  is differentiable in  $(2, 4)$ .

$$\begin{aligned} \text{iii) } f(2) &= (2)^2 - 6(2) + 8 \\ &= 4 - 12 + 8 \\ &= 0. \end{aligned}$$

$$\begin{aligned} f(4) &= (4)^2 - 6(4) + 8 \\ &= 16 - 24 + 8 \\ &= 0. \end{aligned}$$

Clearly  $f(2) = f(4) = 0$ .

Hence,  $f(x)$  satisfies all the conditions of Rolle's theorem.

$\therefore$  There exist at least one point  $c \in (2, 4)$  such that

$$f'(c) = 0$$

$$f'(c) = 2c - 6$$

$$\Rightarrow 2c - 6 = 0$$

$$\Rightarrow 2c = 6$$

$$\Rightarrow c = \frac{6}{2}$$

$$\Rightarrow c = 3 \in (2, 4).$$

Thus, the required value of  $c$  is  $3 \in (2, 4)$ .

Hence, Rolle's theorem is verified. //

2) Verify Rolle's theorem for  $f(x) = (x-a)^m(x-b)^n$  in  $[a, b]$  where  $m$  and  $n$  are positive integers.

Solution: Given  $f(x) = (x-a)^m(x-b)^n$  in  $[a, b]$

i) Since  $f(x) = (x-a)^m(x-b)^n$  is a polynomial function in  $x$ .  $\Rightarrow f(x)$  is continuous for every value of  $x$ .

ii) Derivative  $f'(x) = (x-a)^m n(x-b)^{n-1} + (x-b)^n m(x-a)^{m-1}$   
 $\forall x \in (a, b)$ .

$\Rightarrow f(x)$  is differentiable in  $(a, b)$ .

iii)  $f(a) = (a-a)^m(a-b)^n$   
 $= 0.$

$$f(b) = (b-a)^m(b-b)^n$$
$$= 0.$$

Clearly,  $f(a) = f(b) = 0$ .

Hence,  $f(x)$  satisfies all the conditions of Rolle's theorem.

$\therefore$  There exist at least one point  $c \in (a, b)$  such that

$$f'(c) = 0.$$

$$f'(c) = (c-a)^m n(c-b)^{n-1} + (c-b)^n m(c-a)^{m-1}$$

$$\Rightarrow 0 = (c-a)^{m-1}(c-b)^{n-1} [n(c-a) + m(c-b)]$$

$$\Rightarrow 0 = [n(c-a) + m(c-b)]$$

$$\Rightarrow 0 = nc - na + mc - mb$$

$$\Rightarrow 0 = c[n+m] - [na+mb]$$

$$\Rightarrow na+mb = c[n+m]$$

$$\Rightarrow c = \frac{na+mb}{n+m} \in (a, b).$$

Thus the required value of  $c$  is  $\frac{na+mb}{n+m} \in (a, b)$ .

Hence, Rolle's theorem is verified. //

3) Verify Rolle's theorem for  $(x-2)^2(x-3)^3$  in  $[2, 3]$ .

Solution: Let  $f(x) = (x-2)^2(x-3)^3$  in  $[2, 3]$

i) Since  $f(x) = (x-2)^2(x-3)^3$  is a polynomial function in  $x$ .

$\Rightarrow f(x)$  is continuous for every value of  $x$ .

ii) Derivative  $f'(x) = (x-2)^2 \cdot 3(x-3)^2 + (x-3)^3 \cdot 2(x-2)$   
 $\forall x \in (2, 3)$

$\Rightarrow f(x)$  is differentiable in  $(2, 3)$ .

$$\begin{aligned}\text{iii) } f(2) &= (2-2)^2(x-3)^3 \\ &= 0 \\ f(3) &= (3-2)^2(3-3)^3 \\ &= 0\end{aligned}$$

Clearly,  $f(2) = f(3) = 0$ .

Hence,  $f(x)$  satisfies all the conditions of Rolle's - theorem.

$\therefore$  There exists at least one point  $c \in (2, 3)$  such that

$$f'(c) = 0$$

$$f'(c) = (c-2)^2 \cdot 3(c-3)^2 + (c-3)^3 \cdot 2(c-2)$$

$$(c-2)^2 \cdot 3(c-3)^2 + (c-3)^3 \cdot 2(c-2) = 0$$

$$\Rightarrow (c-2)(c-3)^2 [(c-2)3 + 2(c-3)] = 0$$

$$\Rightarrow (c-2)(c-3)^2 [3c-6+2c-6] = 0$$

$$\Rightarrow 5c-12=0$$

$$\Rightarrow c = 12/5 \in (2, 3).$$

Thus, the required value of  $c$  is  $12/5 \in (2, 3)$ .

Hence, Rolle's theorem is verified. //